Modified Nodal Analysis for MEMS with Multi-Energy Domains

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ABSTRACT

A modified nodal analysis approach for the design, simulation, and display of three-dimensional microelectromechanical systems with coupled energy domains is presented. Static, steady-state, and transient algorithms are introduced followed by examples. Transient and static analysis in noninertial frames is now supported. Simulated results include (1) three-dimensional modal analysis of a torsional micro-mirror, (2) static analysis of a gap-closing actuator with coupled circuit, mechanical, and electrostatic energy domains, (3) the induced current response of a comb-driven multi-mode resonator, and (4) the transient response of a gap-closing actuator due to a time varying voltage showing a nonlinearly varying resonance frequency and displacement.

Keywords: Modified nodal analysis (MNA), static, steady-state, transient, SUGAR, accelerating frames.

1 INTRODUCTION

SUGARv1.0 is a collection of MATLAB [1] routines that take a netlist description of MEMS devices and perform static, steady-state, modal, and transient analyses of three-dimensional mechanical structures and electrical circuits. We're using modified nodal analysis (MNA) like SPICE [2]. Fedder et. al. at CMU take a similar nodal analysis approach [3,4]. SUGARv1.0 code, demo files, presentations, publications, and manual is available from the University of California at Berkeley on the Berkeley Sensor and Actuator Center's website [5].

Our previous version, SUGARv0.5, was only planar-mechanical with very limited electrostatic capability [6]. New features for SUGARv1.0 include the following: (1) Models are stored in standard format in individual files, allowing new models to be created easily. (2) Process data is stored in a separate file. (3) Node and branch variables can be mechanical, electrical, thermal, etc. (4) Rotationally- and translationally-accelerating reference frames allow simulation of accelerometer and gyroscope performance. Three-dimensional model formulations and system matrix assembly follow from previous work done in two dimensions [7] and will not be presented here.

2 ALGORITHMS

2.1 Static Analysis

Static analysis attempts to find a final equilibrium state for a MEMS device with coupled energy domains. The determination of multiple equilibria (e.g. buckling) is not currently implemented in SUGARv1.0. These domains may include mechanical forces, electrostatic forces due to voltages from circuits, forces due to thermal stresses, residual stresses, etc. The equation of static equilibrium of a system due to nonlinear forces of various energy domains is expressed as

$$f(\{q\}) = \{0\} \tag{1}$$

where the vector {q} corresponds to the state variables of the system. It may contain nodal mechanical displacement elements, electrical potential elements, etc [8]. The function f contains the modified nodal equations for each energy domain that represents the particular system. For instance, in the mechanical domain the sum of all forces and moments on each node is zero. Likewise, the sum of all currents flowing in and out of each node is zero for electrical elements.

Since the system in general may contain nonlinear elements such as electrostatic gaps, contact forces, and mechanical beams subject to large deflections, numerical methods for solving nonlinear equations are required. SUGARv1.0 uses a Newton-Raphson method [9].

This solver begins by taking an initial guess of the state of the system $\{q_0\}$. It then tries to reach a solution by the iteration of the following equation

$$\{q_{n+1}\} = \{q_n\} - [J(\{q_n\})]^{-1}\{f(\{q_n\})\} \tag{2}$$

where $J(\{q_n\})$ is the Jacobian matrix for the system.

The iteration proceeds until the following condition is met.

$$\|\{q_{n+1}\} - \{q_n\}\| < \zeta \qquad (3)$$

where ζ is the selected tolerance.

2.2 Steady-state Analysis

For steady-state analysis, the system of nonlinear dynamic differential equations containing multi-energy domains are first linearized at a static equilibrium point. Then the higher order linearized system of ordinary differential equations are converted into a first order system. The resultant linearized first order system is given by

$$\{\dot{x}\} = [A]\{x\} + [B]\{u\} \qquad (4a)$$

$$\{y\} = [C]\{x\} + [D]\{u\} \qquad (4b)$$

where {x} is the system dynamic state variable, {u} is the sinusoidal external excitation, {y} is the system dynamic response. A, B, C, and D are the system, input coupling, output, and feed forward matrices respectively. The solution to Equation 4 is obtained by well-know standard methods [10]. It’s solution provides Bode plots as well as modal analysis.

2.3 Transient Analysis

The transient solver performs a time domain response of a MEMS device, which may contain nonlinear elements and excitations that are functions of, say, time and displacement. Several transient numerical methods are available, whereby speed can be traded for accuracy and long-term stability. These numerical methods include an implicit second order Rosenbrock solver for stiff problems where low accuracy is acceptable, an explicit Runge-Kutta 4-5th order solver for non-stiff systems, an implicit multi-step integration method of varying order for stiff problems requiring higher accuracy, and a simple explicit Euler algorithm.

The system of equations governing the transient examples presented in this manuscript are of the form

$$[M]\{\ddot{q}\} + [D]\{\dot{q}\} + [K]\{q\} = \{F(t, \{q\})\} \qquad (5)$$

The matrix coefficients [M], [D], and [K] are the generalized system matrices, which are analogous to the mass, damping, and stiffness matrices in a purely mechanical system. {F(t,{q})} is the excitation vector for the system. It is a function of time and state.

For modeling generality, a high order ODE is converted to a first order system [10] of the form

$$\{\dot{Q}\} \equiv \frac{d}{dt}\begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} = \{f(t, \{q\})\} \qquad (6)$$

For example, the second order ODE given by Equation 5 may be converted to

$$\{\dot{Q}\} = \begin{Bmatrix} \{\dot{q}\} \\ [M]^{-1}(\{F\} - [K]\{q\} - [D]\{\dot{q}\}) \end{Bmatrix} \qquad (7)$$

The net system excitation {F} may include inertial forces as well as the typical on-chip electrostatic forces. For example, in the pure mechanical case, in the event that the MEMS device is subject to an accelerating substrate, the following excitation terms are added to the right hand side of Equation 5,

$$\{F_{Inertial}\} = -[M]\{A\} - 2[M]\{\omega\}\times\{dq/dt\} - [M]\{d\omega/dt\}\times\{R\} - [M]\{\omega\}\times(\{\omega\}\times\{R\}) \qquad (8)$$

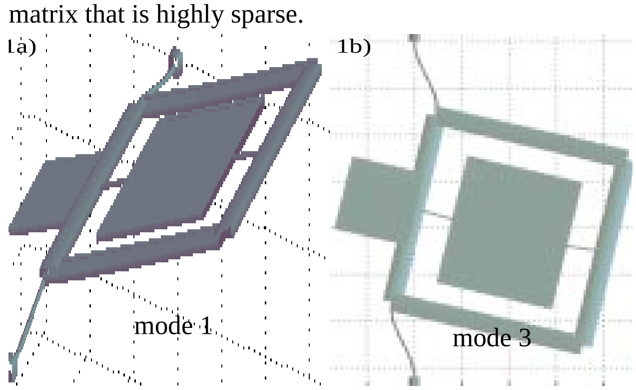
Where R, A, ω, and dω/dt is node position vector, acceleration, angular velocity, and angular acceleration of the substrate, respectively. The terms in Equation correspond to translational, coriolis, transverse, and centrifugal forces respectfully.

3 SIMULATED RESULTS

3.1 Modal Analysis

Figure 1 shows a three-dimensional mechanical mode analysis of a torsional micro-mirror. The mode shapes and frequencies of modes 1, 3, 4 and 6 are shown in Figures 1a through 1d respectively.

The mirror is modeled with only 2-node beam elements. Each node consists of 6 degrees of freedom, {x, y, z, θ_x, θ_y, θ_z}. The system is composed of 12 beam elements, which when assembled into a system forms a [9-(dynamic node) by a 6-(degree of freedom per node)]^2 matrix that is highly sparse.



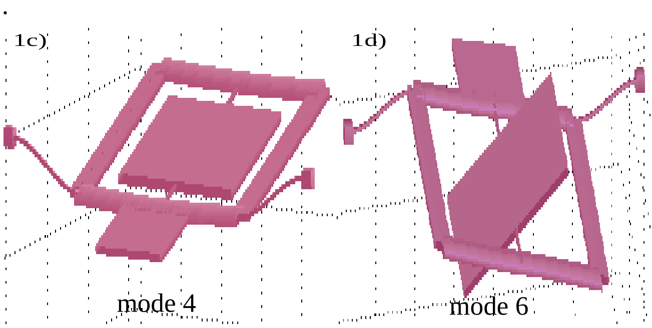


Figure 1: *3D mechanical mode analysis of a torsional mirror. Frequencies of modes 1, 3, 4, and 6 are 15.5kHz, 31.1kHz, 41.7kHz, & 123kHz respectively.*

3.2 Static Simulation

Fig. 2 shows static analysis applied to a gap-closing actuator. The MOS transistor and electrostatic gap are both nonlinear devices, while the three-dimensional mechanical beams are modeled linearly. For the purpose of clarity Fig. 2a depicts circuit elements superimposed on the output display since SUGARv1.0 does not draw circuits at this time. Fig. 2b shows plots of Vout as a function of Vin and displacement as a function of Vin. Simulation correctly calculates the electrical and mechanical operating points.

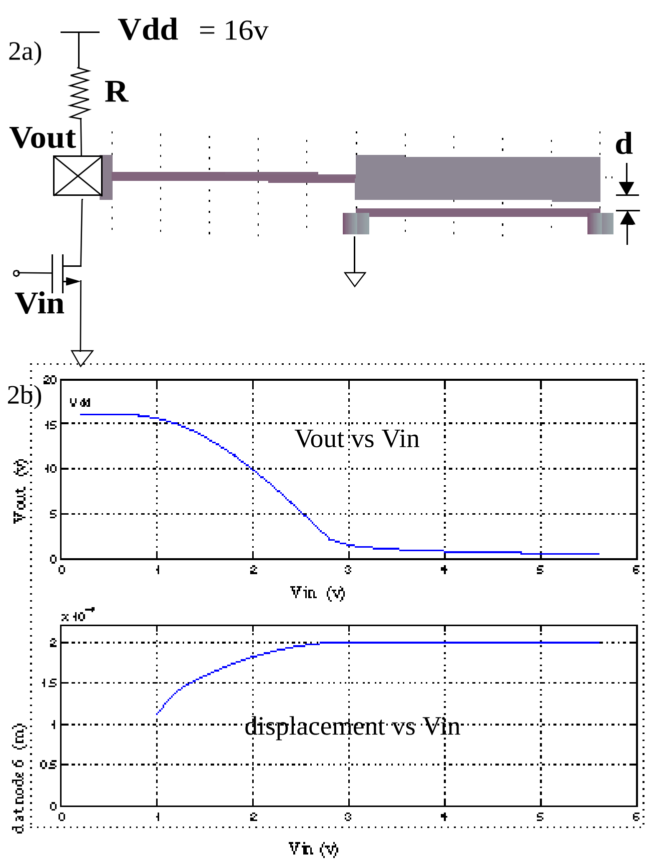


Figure 2: *A DC simulation involving mechanical, electrostatic and circuits analysis coupled together. An invertor circuit is connected to a cantilever beam and electrostatic gap. Pull-in at 15.9V*

3.3 Steady State Response

Fig. 3 demonstrates steady-state analysis applied to a multi-mode resonator. Fig. 3b shows the Bode and phase plot for the current induced on the sensing comb as a function of the frequency of the voltage at the driving comb (see Fig. 3a). The measured modes are with in 5% of experimental frequency given by Brennen et. al. [11].

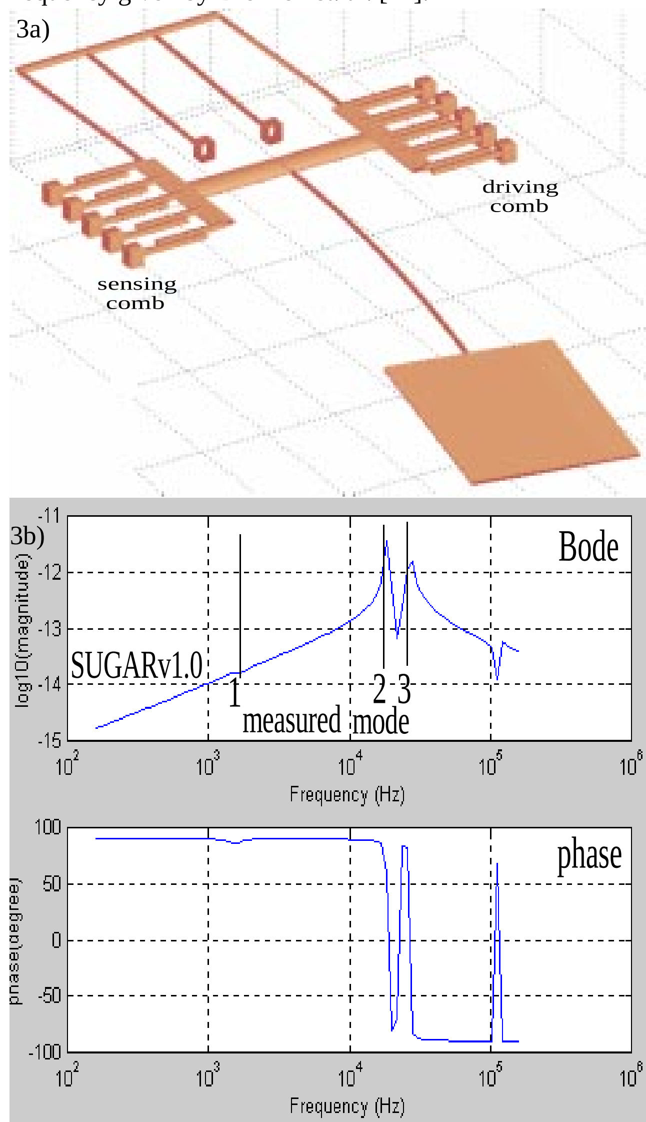


Figure 3: *Simulation of a linear-drive multi-mode resonator. Fig. 3b is a Bode and phase plot of the induced current response of one comb as a function of drive frequency of the other.*

3.4 Transient Dynamics

Fig. 4 shows the transient response of a gap-closing actuator. Fig. 4b shows a plot of displacement as a function of time for the mechanical structure in Fig. 2a. The voltage ramps from 5V at t=5usec to 12V at t=500usec, and then releases, Fig. 4c. As the voltage increases linearly during this time interval, the space between the gap decreases at a nonlinear rate due to electrostatic forces; likewise, the period of oscillation decreases. The amplitude of oscillations decrease exponentially due to the viscous layer of air between the device and substrate. We did not have a squeeze-film damping model at the time of this writing. In contrast to ramping, a lower step voltage of 11.85V will pull-in due to its inertia. Fig. 4d shows the dependence of beam width for a voltage step of 11.75V. Note that inertia, damping, and beam stiffness are functions of width.

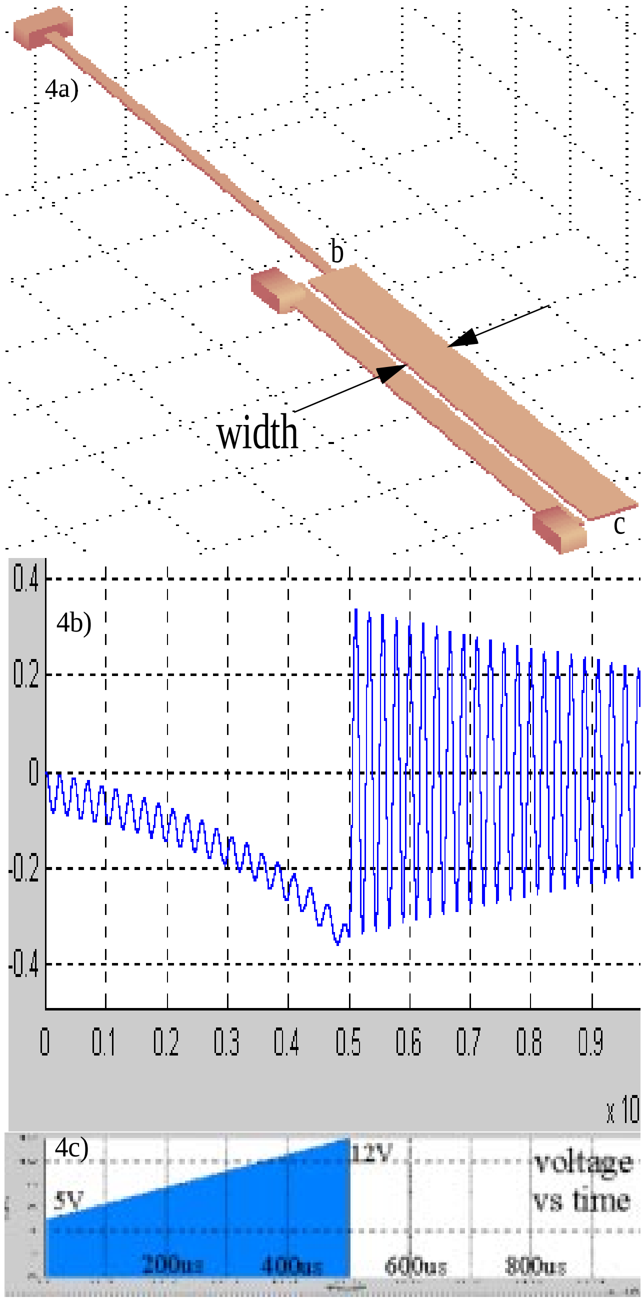


Fig 4: *Transient analysis of a gap-closing actuator. Fig 4b shows the response to a voltage ramp. Fig. 4c shows width dependence subject to a voltage step.*

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